Other Discrete Probability Distribution

1. Discrete Uniform Distribution

A distribution where all outcomes are equally likely

Example: rolling a die once

Formula: $P(x) = \frac{1}{n}$

If Random Variable starts at 1:

Mean: $\frac{N+1}{2}$ Variance: $\frac{N^2-1}{12}$ Standard Deviation: $\sqrt{\frac{N^2-1}{12}}$

Where n : total outcomes

N: Highest Value

If it does not start at 1: $\mu = \frac{a+b}{2}$ $\sigma^2 = \frac{(b-a+1)^2-1}{12}$

Where a is the smallest value and b is the largest value

Example: A small café tracks the number of customers they serve during a slow hour. Based on past data, the number of customers can be:

$$3, 4, 5, 6, \text{ or } 7$$

Each number of customers is **equally likely**.

a. What is the probability of each outcome?
b. What is the expected number of customers per hour?
c. What is the variance?
d. What is the standard deviation?
e. Interpret the given data (hint Mean $\pm$ SD)

2. Bernoulli Distribution

Named after Jacob Bernouli is a probability distribution of a random variable (x) with only two possible outcomes , 1 and 0.

Probability Mass Function

$$P(x) = \begin{cases} P \text{ if } x=1 \\ 1-p, \wedge x=0 \\ 0, \wedge otherwise \end{cases}$$

$$P(X=x) = p^x(1-p)^{1-x}$$

Where:

- $x = 0$ or 1
- p = probability of success

Where: P is success q is failure and q = 1-p

$\mu = p$ $\sigma^2 = p(1-p)$ $\sigma = \sqrt{p(1-p)}$

Example: A student takes a pass/fail quiz.

If the student passes → Success (1)

If the student fails → Failure (0)

The probability of passing is: p=0.7

a. What is the probability of passing?
b. What is the probability of failing?
c. What is the expected value (mean)?
d. What is the variance?

e. What is the standard deviation and its interpreation (hint variability in the outcome)

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$

3. BINOMIAL DISTRIBUTION

Counts the number of successes in several trials

X~Bi (n,p) read as Variable x followed by a binomial distributioin with **n** number of trials and **p** probability success

$\binom{n}{k}$ number of ways to choose k successes from n trials

$$\mu = np \qquad \sigma^2 = np(1-p) \qquad \sigma = \sqrt{np(1-p)}$$

When to use:

- Fixed number of trials (n)
- Two outcomes (success/failure)
- Same probability each trial
- Independent trials

$$P(X=k)=\frac{n!}{k!(n-k)!}\cdot p^k\cdot(1-p)^{n-k}$$

Where n= number of binomial trials

k= number of success among n trials

p = probability of success in one of the n trials

q = probability of failure in one of the n triales

Example:

A student answers a **5-question multiple-choice quiz.**

Each question has 2 choices (correct or wrong).

The probability of getting a correct answer is:

$$p = 0.6$$

Let X = number of correct answers.

Question: What is the probability that the student gets exactly 3 correct answers?

What is the interpretation of standard deviation? (

Note: The standard deviation measures how much the values deviate or vary from the mean. A small SD indicates consistency, while a large SD indicates greater variability."

4. POISSON PROBABILITY DISTRIBUTION

The Poisson distribution is used to model the number of times an event happens in a fixed interval (time, area, space) or "Counts how many times something happens".

$$P(X=k)=\frac{e^{-\lambda}\lambda^k}{k!}$$

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

$$\sigma = \sqrt{\lambda}$$

When to Use:

✔ Events happen randomly

✔ Events are independent

✔ We count occurrences in a fixed interval

✔ The average rate is known

Where:

- λ (lambda) = average number of events
- k = number of occurrences
- $e \approx 2.718$

Problem:

A call center receives 4 calls per minute on average. Find the probability that exactly 2 calls occur in a minute.

5. HYPERGEOMETRIC PROBABILITY DISTRIBUTION

The hypergeometric distribution is used when we are selecting items **WITHOUT replacement** from a group or Choosing items from a group where probabilities change after each pick

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

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Symbol	Meaning
N	total population
K	number of success items
n	number of draws/sample size
k	number of successes chosen

$$\mu = n \cdot \frac{K}{N}$$

$$\sigma^2 = n \cdot \frac{K}{N} \cdot \frac{N-K}{N} \cdot \frac{N-n}{N-1}$$

Problem:

A box contains:5 red balls and 7green balls. You randomly pick 3 balls without replacement.
Find the probability of getting 2 red balls.